

# Course logistics

10%

Assignments

90%

Oral final exam

Assignments become available in the week their material is taught. They are due **before the next class**.

**Please don't use LLMs.** The assignments are self-contained. Together with the course slides, they give you everything you need to complete them.

# What we learned in Lecture 01

1. A **call** gives its holder the right to buy at the strike without requiring exercise.
2. A **put** gives its holder the right to sell at the strike without requiring exercise.
3. At expiry, the holder exercises only when doing so has positive value.
4. The holder pays a premium; the short option can be assigned and must perform.
5. Option-like rights appear in finance, insurance, contracts and everyday life.

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We defined the right. Now we need to value it.

# What is a right without an obligation worth?

You can take the good outcomes.

You can walk away from the bad ones.

## The right

Keep the upside if it happens.

## No obligation

Reject the downside if it happens.

**Key question for today:**

*What determines the value of that asymmetry?*

# Meet Boring Utility Co.

Share price today

**\$100**

A regulated utility with predictable cash flows.

- No major announcements expected
- Few operational surprises
- Three months from now, it will probably be worth about **\$100**.

Very little uncertainty about the outcome.

# Now meet Binary Biotech

Share price today: **\$100** · Drug-trial result arrives in three months.

Possible outcomes:

**Drug fails**

**\$50**

**Drug works**

**\$400**

What probabilities of success and failure are implied by today's \$100 price?

$$100 = q(400) + (1 - q)(50) \implies q = \frac{1}{7} = 14.3\%$$

Success: **14.3%** · Failure: **85.7%**

These are toy pricing probabilities, not a forecast of the trial. Assume zero rates, no dividends and exactly two expiry outcomes.

# Same call. Very different value.

Three-month call · strike  $K = \$100$

## Boring Utility Co.

Almost surely  $S_T = \$100$ :

$$C_0 \approx \max(100 - 100, 0) = \$0$$

No upside surprise to capture.

## Binary Biotech

Using the implied probabilities:

$$C_0 = \frac{1}{7}(400 - 100) + \frac{6}{7}(0)$$

$$C_0 = \$42.86$$

Same spot. Same strike. Same average future stock price. **Different uncertainty.**

Undiscounted toy model. Zero rates and dividends.

# How much does uncertainty add?

Keep the option strike and average expiry share price at \$100. Move the outcomes apart.

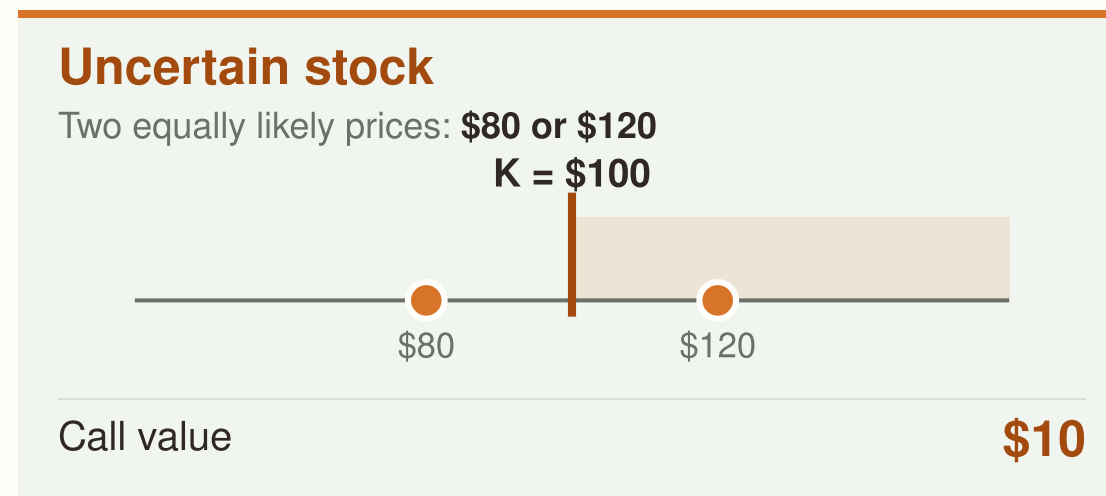
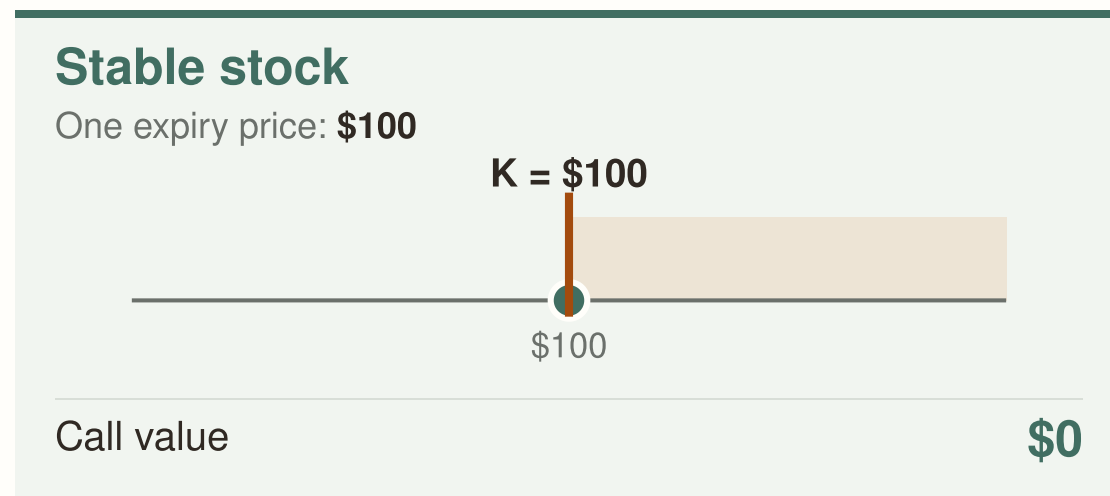


The stock's average is unchanged. Calls and puts gain from opposite tails.

Equal toy pricing probabilities for each two-state case. Zero rates and dividends.

# Same call terms. Two different stocks.

Both stocks trade at \$100 today. One finishes at \$100; the other finishes at \$80 or \$120. Give both calls the **same strike**.



Extra value on the uncertain stock **+\$10**

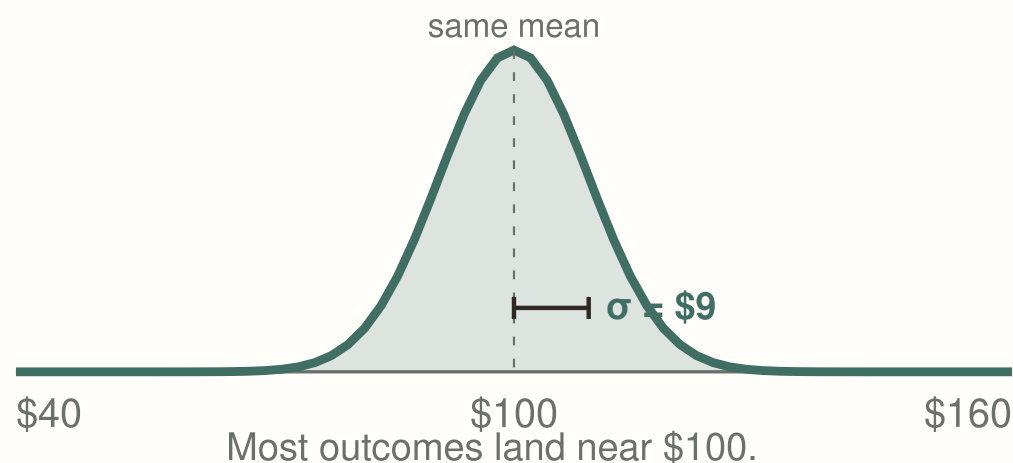
The uncertain stock has an outcome on each side of the strike.

Values are undiscounted expected call payoffs under toy pricing probabilities.

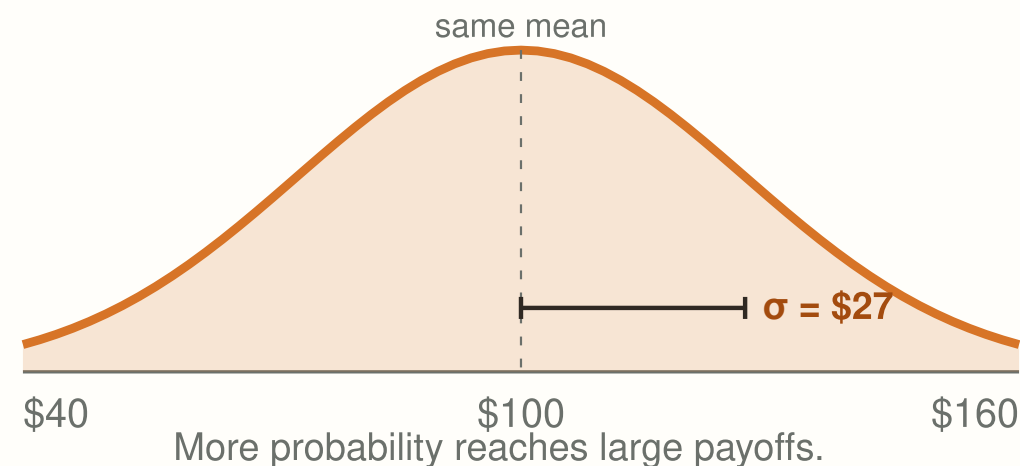
# Uncertainty creates option value

A wider distribution has a larger standard deviation,  $\sigma$ . That extra uncertainty can make a call more valuable even when the mean is unchanged.

## Tight distribution



## Wide distribution



This naturally leads us from thinking about outcomes to thinking about **distributions**.

# Options are bets on distributions

Options **are** bets on volatility. More generally, they are bets on the shape of probability distributions.

- A stock payoff is linear.
- A call payoff has a **kink**: losses stop at zero while gains keep growing.

$$C_T = (S_T - K)^+$$

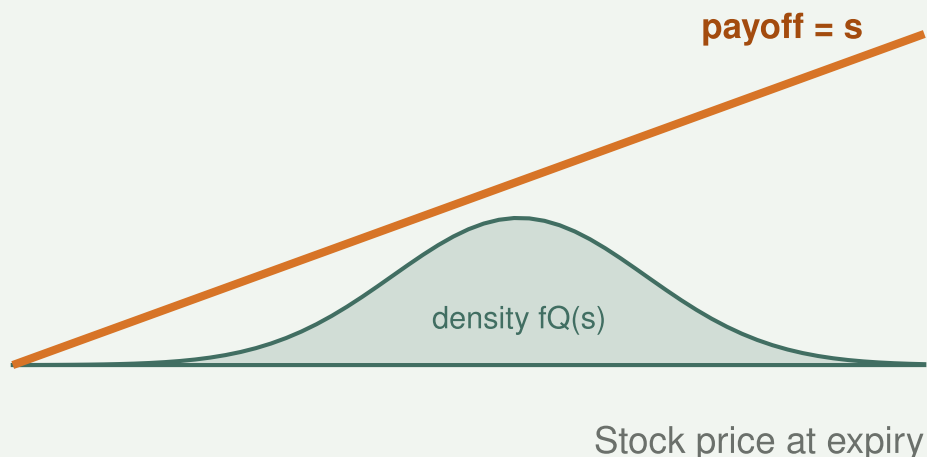
Convex payoff

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This non-linearity makes the *distribution* of outcomes matter, not only their average.

# Expected payoff is an integral

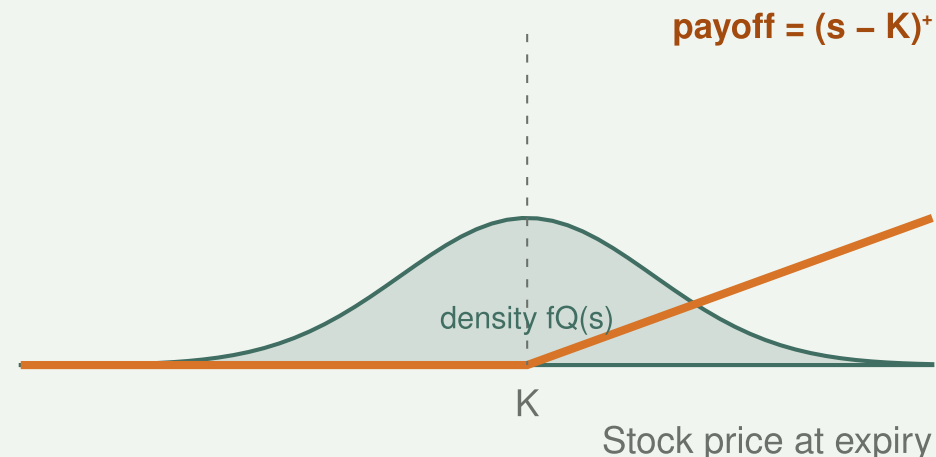
## Stock · linear payoff



$$S_0 = e^{-rT} \int_0^{\infty} s f_Q(s) ds$$

Only the mean matters.

## Call · non-linear payoff



$$C_0 = e^{-rT} \int_K^{\infty} (s - K) f_Q(s) ds$$

The shape of the distribution matters.

Here  $f_Q$  is a **pricing density**, not a forecast. Lecture 04 will explain the distinction.

# Which call would you own?

Both calls have strike \$100. Both distributions have an average expiry price of \$100.

## Distribution A

\$80 or \$120, each with **50%** probability.

What is the expected call payoff?

## Distribution B

\$80 with **25%**, \$100 with **50%**, or \$120 with **25%** probability.

What is the expected call payoff?

**A: \$10. B: \$5.** The range is the same, but A puts more probability on the upside payoff.

Toy pricing probabilities and zero rates. Compare values, not just the highest possible payoff.

# The call is in the money. Exercise now?

So far, we have valued what options may pay **at expiry**. An American call also lets you exercise **before** expiry. Should you?

Suppose the stock is \$110 and the call's strike is \$100.

## Exercise now

Receive a share worth \$110 and pay \$100.

Value now: **\$10**

## Keep the choice

At expiry, the share is either \$80 or \$140, each equally likely.

Call payoffs: **\$0 / \$40**

Waiting is worth **\$20** in this toy model. Exercising now gives up the remaining \$10 of time value.

Toy pricing probabilities, zero rates, no dividends and no trading frictions.

# Why is early exercise usually suboptimal?

An unexpired option contains two things:

## Intrinsic value

What exercise gives you **today**.

+

## Time value

The remaining chance that uncertainty creates something better.

Exercise keeps the intrinsic value but destroys the remaining time value.

If you want to exit, selling the option is usually better than exercising it.

Important exceptions and complications: dividends, stock borrow, interest rates, puts, transaction costs and contract details.

# More time cannot hurt an American option

**ARBITRAGE CONCEPT**

#2

Two otherwise identical American options expire at  $T_1$  and  $T_2$ , with  $T_2 > T_1$ .

**Shorter expiry ·  $T_1$** 

Exercise at any time through  $T_1$ .

**Longer expiry ·  $T_2$** 

Every earlier exercise date, plus more.

$$V_A(T_2) \geq V_A(T_1)$$

The longer-dated American option contains all the rights of the shorter-dated option.

**European options:** more time usually adds value, but this is not pure dominance. The later contract cannot be exercised at  $T_1$ , so dividends, rates and carry can affect the comparison.

# A quieter stock needs more time

Same spot, strike, rates, dividends, and call style. Only daily volatility and expiry differ. Near the money, value roughly scales with total volatility; **variances add**.

## Stock A · quiet

Daily standard deviation **1%**

|                              |                              |                              |                              |
|------------------------------|------------------------------|------------------------------|------------------------------|
| Day 1<br>+ (1%) <sup>2</sup> | Day 2<br>+ (1%) <sup>2</sup> | Day 3<br>+ (1%) <sup>2</sup> | Day 4<br>+ (1%) <sup>2</sup> |
|------------------------------|------------------------------|------------------------------|------------------------------|

Variance accumulated

$$V_A = 4(1\%)^2$$

Total standard deviation

**2%**

## Stock B · volatile

Daily standard deviation **2%**

|                              |
|------------------------------|
| Day 1<br>+ (2%) <sup>2</sup> |
|------------------------------|

One day sets the target.

Variance accumulated

$$V_B = (2\%)^2 = 4(1\%)^2$$

Total standard deviation

**2%**

**Stock A: 4 days of variance**      **Same accumulated variance. Same total volatility.**

**4 months of A ≈ 1 month of B:** same total volatility, same toy-model call value.

Toy model: independent daily moves, constant volatility, matched calls, zero rates/dividends, and equal trading days per month.

# Time and volatility can trade places

In a simple model, the scale of a typical percentage move grows like:

$$\sigma\sqrt{T}$$

## Reference

20% annual volatility · 12 months

$$20\% \times \sqrt{12/12} = 20\%$$


## Try another pair

40% annual volatility · 3 months

$$40\% \times \sqrt{3/12} = 20\%$$


More volatility can offset less time. In this simple model, the move scale is  $\sigma\sqrt{T}$ .

A preview, not a universal pricing rule. Events, jumps and changing volatility complicate this scaling.

# Summary

1. An option is valuable because it preserves the **good outcomes** without requiring the bad ones.
2. With the same average outcome, a **wider distribution** can make an option worth more.
3. A non-linear payoff depends on the **shape of the distribution**, not only its mean.
4. Exercise captures intrinsic value but gives up the option's remaining **time value**.
5. More exercise opportunities cannot reduce value, so a longer-dated **American option** cannot be worth less.

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Uncertainty creates value when you have the right to choose.