

Tutorial 1

1 Market-implied probabilities

- (a) BioTech trades at $S_0 = 100$. At the end of its clinical trial, its share price will be either $S_T = 400$ if the drug passes or $S_T = 50$ if it fails. Assuming the price S_0 is fair, what is the market-implied probability of the test succeeding?
- (b) BioTech must now complete two trial stages. Each stage has the same market-implied pass probability q , and the stages are independent under the pricing measure. The terminal share price depends on both results:

Stage 1	Stage 2	S_T
fail	fail	50
fail	pass	80
pass	fail	120
pass	pass	400

The shares trade at $S_0 = 100$.

- (i) Draw the two-stage event tree and assign a probability to each terminal outcome.
- (ii) Determine q and compute the market-implied probability of each of the four terminal outcomes.

2 Easy derivatives

Compute the following derivatives.

(a) $\frac{d \tan(x)}{dx}$.

(b) $\frac{d \sin(x^2)}{dx}$.

(c) $\frac{d}{dx} \sqrt{\frac{x+1}{x-1}}$.

3 Expected values

Compute the following integrals over $\mathbb{R}$.

(a) $\int_{\mathbb{R}} x e^{-x^2} dx$.

(b) $\int_{\mathbb{R}} \frac{2x}{x^2 + 3x + 2} dx$.

(c) $\int_{\mathbb{R}} \frac{e^x}{1 + e^{2x}} dx$.

4 Option payoff

Compute the following integrals, where $\sigma > 0$.

(a) $\int_{\mathbb{R}} x^+ \frac{3}{x(5+2x)} dx$.

(b) $\int_{\mathbb{R}} x^+ \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} dx$.